Rewarding Learning

ADVANCED
General Certificate of Education
2025

Centre Number

Candidate Number

Mathematics

Assessment Unit A2 1
assessing
Pure Mathematics

[AMT11]

AMT11

THURSDAY 29 MAY, MORNING

TIME

2 hours 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number in the spaces provided at the top of this page.
You must answer **all twelve** questions in the spaces provided.
Do not write outside the boxed area on each page or on blank pages or tracing paper.
Complete in black ink only. **Do not write with a gel pen**.
Questions which require drawing or sketching should be completed using an HB pencil.
Show clearly the full development of your answers. **Answers without working may not gain full credit.**
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 150
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

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1 List the terms of the following sequences for $n = 1, 2, 3, 4$

State whether each sequence diverges, converges or oscillates.

(i) $\dfrac{2n^2}{3+n}$ [2]

(ii) $\dfrac{1-2n}{3n}$ [2]

[Turn over

2 The 4th and 9th terms in an arithmetic progression are 89 and 69 respectively.

(i) Find the first term and the common difference for this arithmetic progression. [5]

(ii) In this arithmetic progression, the last term is the only negative term.

Find the number of terms in the progression. [4]

3 **Fig. 1** below shows the graph of $y = f(x)$

The points P(0, 4) and Q(2, 2) are labelled as shown.

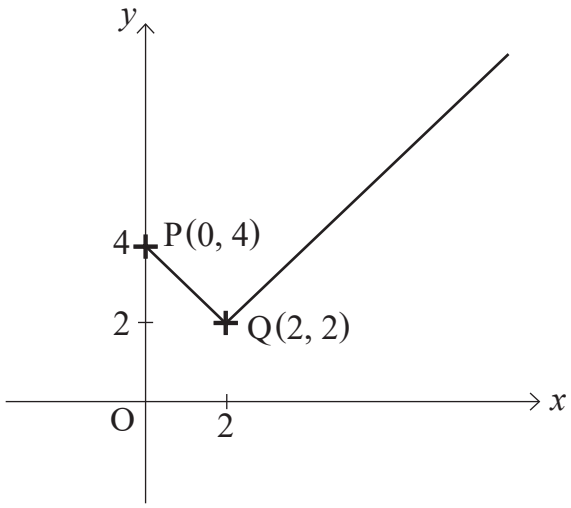
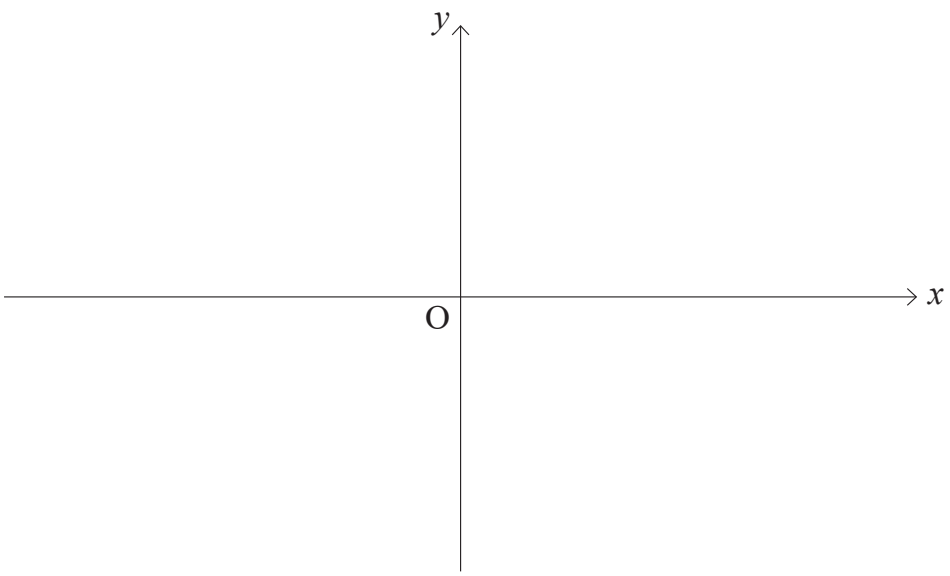


Fig. 1

(i) On the axes below, sketch the graph of

$$y = 1 + f\left(\frac{x}{2}\right)$$

Clearly label the images of the points P and Q.

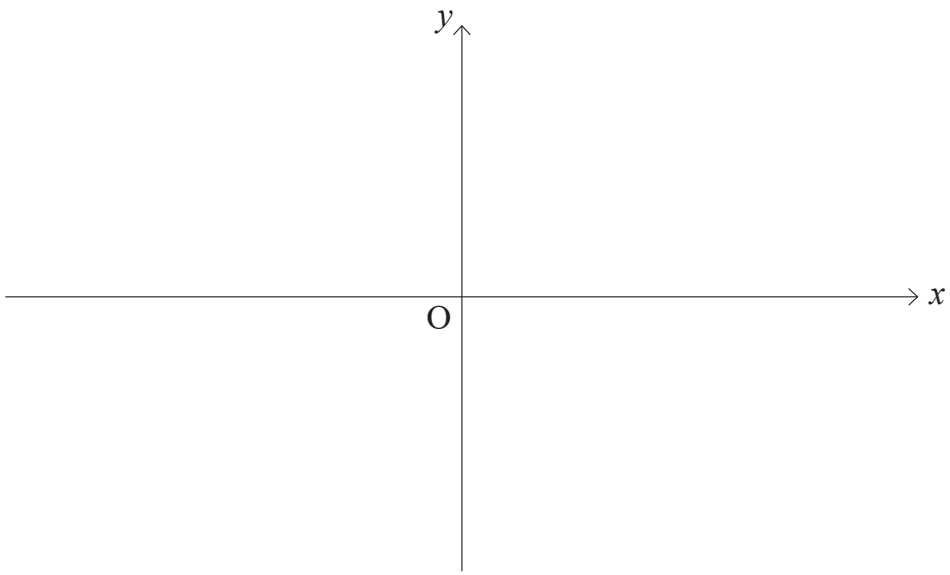


[3]

(ii) On the axes below, sketch the graph of

$$y = 4 - f(x)$$

Clearly label the images of the points P and Q.



[3]

[Turn over

4 Ella wants to make party hats for her son's birthday party.

She creates a template by cutting a major sector AOB from circular card.

The card has centre O and radius 10 cm.

The major arc AB subtends an angle θ radians at O, as shown in **Fig. 2** below.

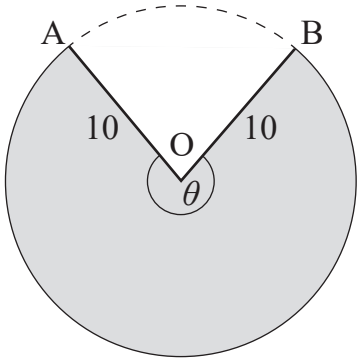


Fig. 2

She will then fold the cards into cones before decorating them.

The template must have a major arc length of 14π cm.

(i) Find the exact value of θ that Ella should use when creating her sector template. [2]

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(ii) Find the area of card wasted in creating this template. [4]

5 **(a)** On the axes below, sketch the graph of $y = \cot\theta$ for $0° \leqslant \theta \leqslant 360°$

[3]

(b) **(i)** Prove the identity

$$2\cot 2\theta \equiv \cot\theta - \tan\theta$$ [4]

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(ii) Hence, or otherwise, find the solutions of

$$\cot\theta - \tan\theta = -2 \quad \text{for} \quad 0° \leqslant \theta \leqslant 360° \qquad [5]$$

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6 **(a)** Differentiate the function $f(x)$ with respect to x, where $f(x)$ is defined as

$$f(x) = e^{2x+1} \operatorname{cosec} 3x \qquad [4]$$

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(b) A curve is given by the equation

$$2x + \frac{5x^2}{y^3} = 3$$

Find the equation of the tangent to the curve at the point (–1, 1).

Leave your answer in the form $ax + by + c = 0$, where a, b and c are integers. [9]

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7 **(i)** Show that

$$\int \sin^2 x \, dx = \frac{1}{2}x - \frac{1}{2}\sin x \cos x + c \qquad [5]$$

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(Questions continue overleaf)

(ii) A vase is modelled by rotating the curve

$$y = 2 - \sin x$$

between the lines $x = 0$ and $x = 2\pi$ through 2π radians about the x-axis, as shown in **Fig. 3** below.

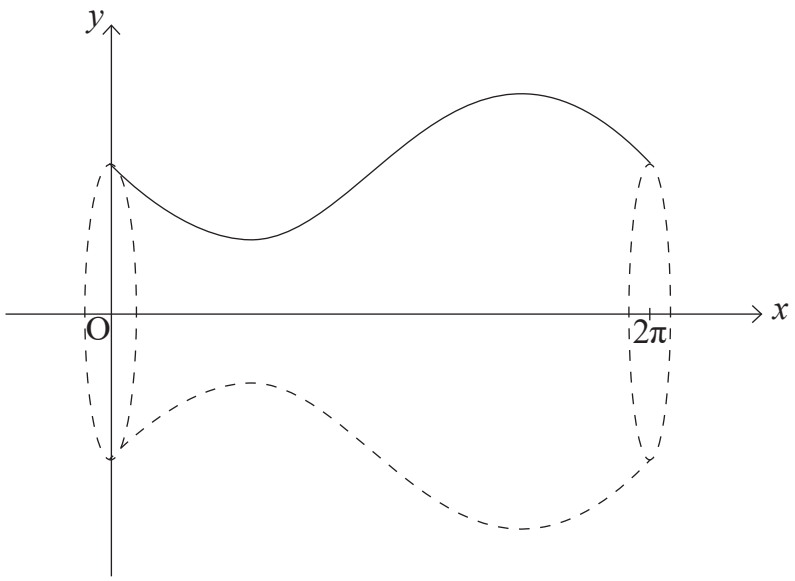

Fig. 3

Using the answer shown in **(i)**, or otherwise, find the exact volume of the vase. [7]

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8 The graph of the function $f(x) = 3(1 + \ln x)^2$ is shown in **Fig. 4** below.

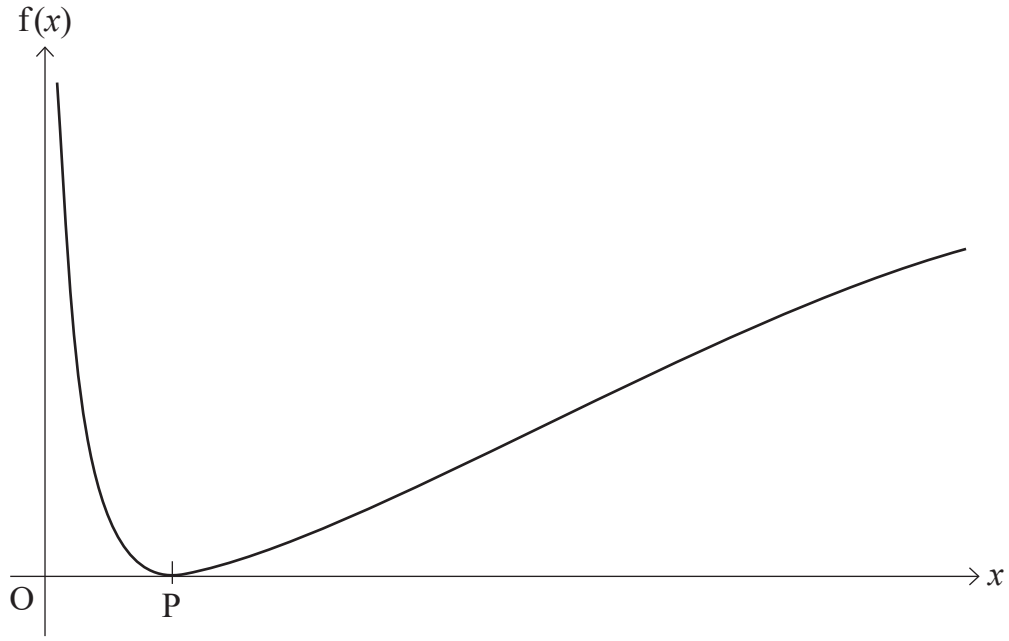

Fig. 4

The stationary point P lies on the x-axis.

(i) Using differentiation, find the exact value of the x coordinate of P.

Confirm that P is a minimum turning point. [9]

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(ii) State the equation of the vertical asymptote of the function f(x). [1]

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(iii) State the domain and range of the function f(x). [2]

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f(x) can be considered as a composite function such that f(x) = hg(x), where

$$g(x) = \ln x - 1 \quad x \in \mathbb{R}, \quad x > 0$$

and h(x) has domain $x \in \mathbb{R}$

(iv) Write down the function h(x). [1]

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(v) Find the inverse function $g^{-1}(x)$, stating its domain. [4]

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9 **(i)** Use the Trapezium Rule with 4 strips to find an approximate value for

$$\int_0^1 \frac{2}{5+\sqrt{x}}\,\mathrm{d}x \qquad [5]$$

(ii) Using the substitution $u = 5 + \sqrt{x}$, find the value of

$$\int_0^1 \frac{2}{5 + \sqrt{x}} \, dx$$ [8]

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(iii) Explain how the use of the Trapezium Rule in **(i)** could be modified to obtain a better approximation to the integral

$$\int_0^1 \frac{2}{5+\sqrt{x}}\,\mathrm{d}x$$ [1]

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10 (a) The expression $2\sqrt{2}\cos x + \sin x$ can be written in the form $R\cos(x - \alpha)$, where R is an integer and $0° < \alpha < 90°$

(i) Find the values of R and α. [6]

(ii) Hence determine the minimum value of

$$3 + 4\sqrt{2}\cos x + 2\sin x$$ [3]

(b) A toy car is moving around a circular track such that its coordinates relative to a fixed origin O are given parametrically by

$$x = 3\cos t + 4\sin t + 2 \qquad \text{and} \qquad y = 4\cos t - 3\sin t - 1$$

where t is the time in seconds.

(i) By using parametric differentiation, show that

$$\frac{dy}{dx} = \frac{2 - x}{y + 1} \qquad [5]$$

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Initially, the car was at the point with coordinates (5, 3).

(ii) By solving the differential equation shown in **(i)**, find the equation of the track. [7]

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11 (i) Write the following expression in partial fractions.

$$\frac{4x^2 + 11x + 7}{(1 + 2x)(3 + x)}$$ [9]

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[Turn over

(ii) Hence, using the binomial theorem, expand

$$\frac{4x^2 + 11x + 7}{(1 + 2x)(3 + x)}$$

in ascending powers of x, up to and including the term in x^2 [8]

[Turn over

12 The graphs of

$$y = \ln x, \quad y = x - 2 \quad \text{and} \quad x = 1$$

are shown in **Fig. 5** below.

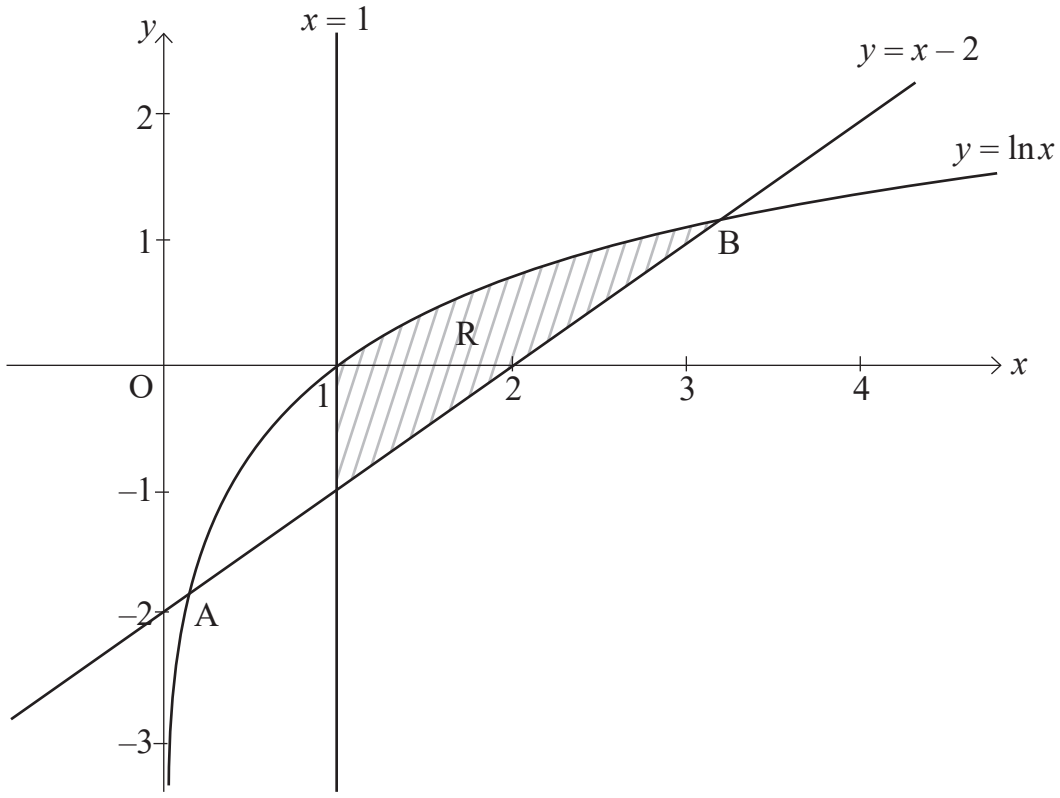

Fig. 5

The graphs of $y = \ln x$ and $y = x - 2$ intersect at the points A and B.

Point B has an x coordinate close to 3

(i) Show that the x coordinate of B lies between $x = 3$ and $x = 4$ [4]

(ii) Starting with $x_0 = 3$, use the Newton-Raphson method once to find a better approximation for the x coordinate of B.

Give your answer correct to three decimal places. [5]

The shaded region R, as shown in **Fig. 5**, is the area enclosed between the two lines and the curve.

(iii) Using the approximation you have found in **(ii)**, find the area of R. [10]

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Question Number	**Marks**
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Total Marks	

Examiner Number